

## How to use Agent X's error checking system

Agent X wants to send the message  $abcd$ , where each letter represents 0 or 1.

She works out:

- a check digit  $x$  (either 0 or 1) so there are an even number of 1s in  **$xabd$**
- a check digit  $y$  (either 0 or 1) so there are an even number of 1s in  **$yacd$**
- a check digit  $z$  (either 0 or 1) so there are an even number of 1s in  **$zbcd$**

Agent X then transmits the message  **$xyazbcd$** .

The base camp can check whether the message they receive has been transmitted correctly by counting the number of 1s in  $xabd$ ,  $yacd$  and  $zbcd$ .

If these are all even numbers, the message has been transmitted correctly, and they can read off the message Agent X sent by reading the 3rd, 5th, 6th and 7th digits.

If one or more of these strings  $xabd$ ,  $yacd$ ,  $zbcd$  contains an odd number of 1s, the base camp can work out which digit has been altered in transmission.

**For example**, suppose  $b$  has been altered in transmission:

**$xabd$**  and  **$zbcd$**  will contain an odd number of 1s.

The fact that  **$yacd$**  has an even number of 1s in it means  $y$ ,  $a$ ,  $c$  and  $d$  must have been transmitted correctly.

The only digit that is in both  $xabd$  and  $zbcd$ , but not in  $yacd$ , is  $b$ , so they can deduce that  $b$  is the incorrect digit.

They can then correct the error and read off the message Agent X transmitted.

**Another example:** suppose base camp receives the message 0100111.

$x$	$y$	$a$	$z$	$b$	$c$	$d$
0	1	0	0	1	1	1

$xabd$ : Even

$yacd$ : Odd

$zbcd$ : Odd

This means that  $x$ ,  $a$ ,  $b$  and  $d$  are all correct, one of  $y$ ,  $a$ ,  $c$  and  $d$  is incorrect, and one of  $z$ ,  $b$ ,  $c$  and  $d$  is incorrect. The digits that appear in both  $yacd$  and  $zbcd$  are  $c$  and  $d$ , but we know  $d$  is correct, so  $c$  must be the incorrect digit. So the message should have read 0100101, so Agent X sent 0101, or North-North-West.