

Agent X wants to send the message $abcd$, where each letter represents 0 or 1.

She works out:

- a check digit x (either 0 or 1) so there are an even number of 1s in **$xabd$**
- a check digit y (either 0 or 1) so there are an even number of 1s in **$yacd$**
- a check digit z (either 0 or 1) so there are an even number of 1s in **$zbcd$**

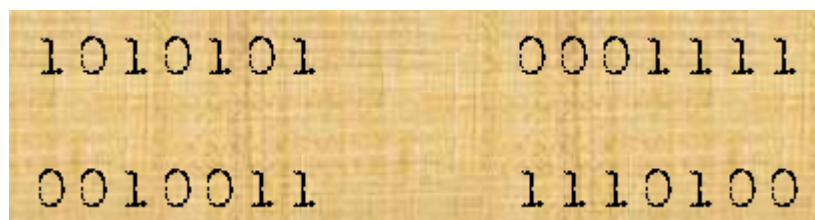
Agent X then transmits the message **$xyazbcd$** .

For example, if Agent X wants to send 0110, she calculates that $x=1$, $y=1$ and $z=0$. She then transmits 1100110.

If you receive Agent X's message, how can you check whether there is an error?

If you find that there is an error, is it possible to recreate the original message?

Here are the messages that Agent X sent using the new system. Again, you can assume that there is at most one altered digit.



1010101	0001111
0010011	1110100

Which ones have been transmitted error-free?

Can you correct the incorrect messages?

**Is it always possible to correct a message that contains an error?
How do you know?**