

Gifted and Talented after school club — Y7

Woolwich Polytechnic

Because we were told not to do the number calculations, we considered how we might use algebra to help.

Firstly, Ayobami decided to use nth term to solve the question and find a solution. He started seeing patterns in the nth term calculations, and realised that some of the answers were the same, before deciding a different approach.

All the children started using algebra on the grid by replacing the four corners with a,b,c and d, like so:

A ↑ B

C D

With that, we were able to carry out each rotation/flip with another piece of paper exactly like the one above. After we all worked out each rotation/flip combination using letters, Ayobami decided to replace the letters with numbers, like so:

1 6

31 36

He was able to get a total for each rotation and found the lowest and highest ways of rotation/flip. Please read the adjoining file to see the solution and the answer.

Solution:

Consider the four corners of the grid as:

A

B

C

D

where $A < B < C < D$

After each turn, these are the algebraic calculations when the letter on the top and bottom are multiplied.

- a) 360° : $a^2 + b^2 + c^2 + d^2$
- b) 90° clockwise: $a(c+b) + d(b+c)$
- c) 180° : $2ad + 2bc$
- d) 270° clockwise: $a(b+c) + d(b+c)$
- e) Flip the sheet: $2ab + 2cd$
- f) Flip and 90° clockwise: $b^2 + 2ad + c^2$
- g) Flip and 180° : $2ac + 2bd$
- h) Flip and 270° : $2bc + a^2 + d^2$

We can see that option b and d are the same.

(cont.)

Substituting the numbers 1,6,31 and 36 in place of a,b,c and d respectively, we get:

$$360^\circ: a^2 + b^2 + c^2 + d^2$$

$$(1 \times 1) + (6 \times 6) + (31 \times 31) + (36 \times 36) = \underline{2294}$$

$$90^\circ \text{ clockwise: } a(c+b) + d(b+c)$$

$$1(31+6) + 36(31+6) = \underline{1369}$$

$$180^\circ: 2ad + 2bc$$

$$2(1 \times 36) + 2(6 \times 31) = \underline{444}$$

$$270^\circ \text{ clockwise: } a(b+c) + d(b+c)$$

$$1(6+31) + 36(6+31) = \underline{1369}$$

$$\text{Flip the sheet: } 2ab + 2cd$$

$$2(1 \times 6) + 2(31 \times 36) = \underline{2244}$$

$$\text{Flip and } 90^\circ \text{ clockwise: } b^2 + 2ad + c^2$$

$$6^2 + 2(1 \times 36) + 31^2 = \underline{1069}$$

$$\text{Flip and } 180^\circ \text{ turn: } 2ac + 2bd$$

$$2(1 \times 31) + 2(6 \times 36) = \underline{494}$$

$$\text{Flip and } 270^\circ \text{ clockwise: } 2bc + a^2 + d^2$$

$$2(6 \times 31) + 1^2 + 36^2 = \underline{1669}$$

Answer: When the letters are replaced with numbers, a 360° turn gives the highest flip/turn combination while an 180° turn gives the lowest total. Since $a < b < c < d$ in all squares within the grid this will be true for the grid as a total.