1. Simply expanding $(1-x)(1+x+x^2+x^3)$:
   $(1-x)(1+x+x^2+x^3)$ = $1+x+x^2+x^3-x-x^2-x^3-x^4$ = $1 - x^4$

   Due to the terms cancelling out, we can see that the only terms remaining leave us with the generalized equation:
   $(1-x)(1+x+x^2+x^3+\cdots x^n)$ = $(1 - x^{n+1})$

2. Using the difference of two squares where $(a+b)(a-b)=a^2-b^2$:
   $(1-x)(1+x)(1+x^2)(1+x^4)$ =
   $(1-x^2)(1+x^2)(1+x^4)$ =
   $(1-x^4)(1+x^4)$ =
   $(1-x^8)$

   Therefore when generalizing, because all the brackets can be factorized, the equation becomes:
   $(1-x)(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n})$ = $1-x^{2n+1}$

3. This requires the double angle formula where $\sin 2A = 2\sin A\cos A$. First rearrange the equation and then simplify:
   $16\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\cos\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right)$ =
   $8\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\sin\left(\frac{x}{8}\right)$ =
   $4\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\sin\left(\frac{x}{4}\right)$ =
   $2\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$ =
   $\sin x\ \therefore$
   $16\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\cos\left(\frac{x}{16}\right)\sin\left(\frac{x}{16}\right) = \sin x\ \therefore$
   $\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\cos\left(\frac{x}{16}\right)$ = $\frac{\sin x}{16\sin\frac{x}{16}}$

   Generalizing, we can see that we can always factorize the expression using the double angle rule in the same way. Therefore:
   $\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\cos\left(\frac{x}{16}\right)\ldots\cos\left(\frac{x}{2^n}\right)$ = $\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}$