# NRICH MATHS CHALLENGE – Three Neighbours

**Rohan**
**Wilson's School**

## 3 Consecutive Numbers:

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | + | 1 | + | 2 | = | 3 |
| 1 | + | 2 | + | 3 | = | 6 |
| 2 | + | 3 | + | 4 | = | 9 |
| 3 | + | 4 | + | 5 | = | 12 |
| 4 | + | 5 | + | 6 | = | 15 |
| 5 | + | 6 | + | 7 | = | 18 |
| 6 | + | 7 | + | 8 | = | 21 |
| 7 | + | 8 | + | 9 | = | 24 |
| 8 | + | 9 | + | 10 | = | 27 |
| 9 | + | 10 | + | 11 | = | 30 |
| 10 | + | 11 | + | 12 | = | 33 |
| 11 | + | 12 | + | 13 | = | 36 |
| 12 | + | 13 | + | 14 | = | 39 |
| 13 | + | 14 | + | 15 | = | 42 |
| 14 | + | 15 | + | 16 | = | 45 |
| 15 | + | 16 | + | 17 | = | 48 |
| - | - | - | - | - | - | - |
| 80 | + | 81 | + | 82 | = | 243 |
| 81 | + | 82 | + | 83 | = | 246 |
| 82 | + | 83 | + | 84 | = | 249 |
| 83 | + | 84 | + | 85 | = | 252 |
| 84 | + | 85 | + | 86 | = | 255 |
| 85 | + | 86 | + | 87 | = | 258 |
| - | - | - | - | - | - | - |
| 90 | + | 91 | + | 92 | = | 273 |
| - | - | - | - | - | - | - |
| 96 | + | 97 | + | 98 | = | 291 |
| 97 | + | 98 | + | 99 | = | 294 |
| 98 | + | 99 | + | 100 | = | 297 |
| 99 | + | 100 | + | 101 | = | 300 |
| 100 | + | 101 | + | 102 | = | 303 |

When I started looking at sets of 3 consecutive numbers, I noticed all the totals were multiples of 3.

I also noticed that if you took the total and divided it by the middle number in the set, you always got an answer of 3. The total was not divisible, in most cases, by either the first or third number in the set.

However, I was able to identify 3 exceptions to this rule. In addition to the total being divisible by the middle value, if the first or third value is 1 or 3, then the total can also be divided by this. This only happens where the total is 6 and 12 (see table).

In order to convince mathematicians, let's consider the first consecutive number to be n,

$n + (n + 1) + (n + 2) = 3n + 3$

3n will always be a multiple of 3 (as long as n is a whole number). Furthermore, when we add 3, the total will remain a multiple of 3.

## 5 Consecutive Numbers:

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| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | + | 1 | + | 2 | + | 3 | + | 4 | = | 10 |
| 1 | + | 2 | + | 3 | + | 4 | + | 5 | = | 15 |
| 2 | + | 3 | + | 4 | + | 5 | + | 6 | = | 20 |
| 3 | + | 4 | + | 5 | + | 6 | + | 7 | = | 25 |
| 4 | + | 5 | + | 6 | + | 7 | + | 8 | = | 30 |
| 5 | + | 6 | + | 7 | + | 8 | + | 9 | = | 35 |
| 6 | + | 7 | + | 8 | + | 9 | + | 10 | = | 40 |
| 7 | + | 8 | + | 9 | + | 10 | + | 11 | = | 45 |
| 8 | + | 9 | + | 10 | + | 11 | + | 12 | = | 50 |
| 9 | + | 10 | + | 11 | + | 12 | + | 13 | = | 55 |
| 10 | + | 11 | + | 12 | + | 13 | + | 14 | = | 60 |
| 11 | + | 12 | + | 13 | + | 14 | + | 15 | = | 65 |
| 12 | + | 13 | + | 14 | + | 15 | + | 16 | = | 70 |
| 13 | + | 14 | + | 15 | + | 16 | + | 17 | = | 75 |
| 14 | + | 15 | + | 16 | + | 17 | + | 18 | = | 80 |
| 15 | + | 16 | + | 17 | + | 18 | + | 19 | = | 85 |
| - | - | - | - | - | - | - | - | - | - | - |
| 80 | + | 81 | + | 82 | + | 83 | + | 84 | = | 410 |
| 81 | + | 82 | + | 83 | + | 84 | + | 85 | = | 415 |
| 82 | + | 83 | + | 84 | + | 85 | + | 86 | = | 420 |
| 83 | + | 84 | + | 85 | + | 86 | + | 87 | = | 425 |
| 84 | + | 85 | + | 86 | + | 87 | + | 88 | = | 430 |
| 85 | + | 86 | + | 87 | + | 88 | + | 89 | = | 435 |
| - | - | - | - | - | - | - | - | - | - | - |
| 90 | + | 91 | + | 92 | + | 93 | + | 94 | = | 460 |
| - | - | - | - | - | - | - | - | - | - | - |
| 96 | + | 97 | + | 98 | + | 99 | + | 100 | = | 490 |
| 97 | + | 98 | + | 99 | + | 100 | + | 101 | = | 495 |
| 98 | + | 99 | + | 100 | + | 101 | + | 102 | = | 500 |
| 99 | + | 100 | + | 101 | + | 102 | + | 103 | = | 505 |
| 100 | + | 101 | + | 102 | + | 103 | + | 104 | = | 510 |

When I started looking at sets of 5 consecutive numbers, I noticed all the totals were multiples of 5.

I also noticed that if you took the total and divided it by the middle number in the set (the 3rd consecutive number), you always got an answer of 5.

In some instances, the total was also divisible by other consecutive numbers in the set, and these have also been highlighted in green in the table. This tended to happen with values of 10 or less where they were factors of the total value.

Using algebra for 5 consecutive numbers:

n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10

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## 7 Consecutive Numbers:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | + | 1 | + | 2 | + | 3 | + | 4 | + | 5 | + | 6 | = | 21 |
| 1 | + | 2 | + | 3 | + | 4 | + | 5 | + | 6 | + | 7 | = | 28 |
| 2 | + | 3 | + | 4 | + | 5 | + | 6 | + | 7 | + | 8 | = | 35 |
| 3 | + | 4 | + | 5 | + | 6 | + | 7 | + | 8 | + | 9 | = | 42 |
| 4 | + | 5 | + | 6 | + | 7 | + | 8 | + | 9 | + | 10 | = | 49 |
| 5 | + | 6 | + | 7 | + | 8 | + | 9 | + | 10 | + | 11 | = | 56 |
| 6 | + | 7 | + | 8 | + | 9 | + | 10 | + | 11 | + | 12 | = | 63 |
| 7 | + | 8 | + | 9 | + | 10 | + | 11 | + | 12 | + | 13 | = | 70 |
| 8 | + | 9 | + | 10 | + | 11 | + | 12 | + | 13 | + | 14 | = | 77 |
| 9 | + | 10 | + | 11 | + | 12 | + | 13 | + | 14 | + | 15 | = | 84 |
| 10 | + | 11 | + | 12 | + | 13 | + | 14 | + | 15 | + | 16 | = | 91 |
| 11 | + | 12 | + | 13 | + | 14 | + | 15 | + | 16 | + | 17 | = | 98 |
| 12 | + | 13 | + | 14 | + | 15 | + | 16 | + | 17 | + | 18 | = | 105 |
| 13 | + | 14 | + | 15 | + | 16 | + | 17 | + | 18 | + | 19 | = | 112 |
| 14 | + | 15 | + | 16 | + | 17 | + | 18 | + | 19 | + | 20 | = | 119 |
| 15 | + | 16 | + | 17 | + | 18 | + | 19 | + | 20 | + | 21 | = | 126 |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 80 | + | 81 | + | 82 | + | 83 | + | 84 | + | 85 | + | 86 | = | 581 |
| 81 | + | 82 | + | 83 | + | 84 | + | 85 | + | 86 | + | 87 | = | 588 |
| 82 | + | 83 | + | 84 | + | 85 | + | 86 | + | 87 | + | 88 | = | 595 |
| 83 | + | 84 | + | 85 | + | 86 | + | 87 | + | 88 | + | 89 | = | 602 |
| 84 | + | 85 | + | 86 | + | 87 | + | 88 | + | 89 | + | 90 | = | 609 |
| 85 | + | 86 | + | 87 | + | 88 | + | 89 | + | 90 | + | 91 | = | 616 |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 90 | + | 91 | + | 92 | + | 93 | + | 94 | + | 95 | + | 96 | = | 651 |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 96 | + | 97 | + | 98 | + | 99 | + | 100 | + | 101 | + | 102 | = | 693 |
| 97 | + | 98 | + | 99 | + | 100 | + | 101 | + | 102 | + | 103 | = | 700 |
| 98 | + | 99 | + | 100 | + | 101 | + | 102 | + | 103 | + | 104 | = | 707 |
| 99 | + | 100 | + | 101 | + | 102 | + | 103 | + | 104 | + | 105 | = | 714 |
| 100 | + | 101 | + | 102 | + | 103 | + | 104 | + | 105 | + | 106 | = | 721 |

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With sets of 7 consecutive numbers, all the totals were multiples of 7.

If you took the total and divided it by the middle number in the set (the $5^{th}$ consecutive number), you always got an answer of 7.

Like the sets of 5 consecutive numbers, there were instances where the total was also divisible by other consecutive numbers in the set, and these have been highlighted in green in the table. This tended to happen with values of 7 or less where they were factors of the total value.

Using algebra for 7 consecutive numbers:

$n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6) = 7n + 21$