Factors, Multiples and Primes

Age 14+ Level ★★★
Worksheet 1

1. **Three Primes**
   How many sets of three prime numbers have the property that the
   product of the three numbers is exactly five times their sum?
   (The order of the three numbers is not important).

2. **Factorised Factorial**
   For a positive integer n, we define $n!$ to be the product of all the
   positive integers from 1 to n; that is $n! = 1 \times 2 \times 3 \times \ldots \times n$.

   If $n! = 2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$, what is the value of n?

3. **Factor List**
   When Tina chose a number N and wrote down all of its factors, apart
   from 1 and N, she noticed that the largest of the factors in the list
   was 45 times the smallest factor in the list.

   How many numbers N could Tina have chosen for which this is the
   case?

4. **Primes and Six**
   Let p and q be prime numbers with $q = p + 2$ and p greater than 3.

   Prove that $pq + 1$ is divisible by 36.

5. **Leftovers**
   As n takes each positive integer value in turn (that is, $n = 1$, $n = 2$,
   $n = 3$...) how many different values are obtained for the remainder
   when n^2 is divided by $n + 4$?

6. **Square Product**
   What is the smallest integer n such that the product

   $$(2^2 - 1)(3^2 - 1)(4^2 - 1)\ldots(n^2 - 1)$$

   is a perfect square?

These problems are adapted from UKMT (ukmt.org.uk) and WMC (competition.ac) problems.