Age 14+ Level ★★★
Worksheet 2

1. **Adding a Square to a Cube**
   Find all 9 integer values of n between 1 and 100 for which $n^2 + n^3$ is a square number.

2. **Fortunate Inflation**
   The price of an item in pounds and pence is increased by 4%.
   The new price is exactly n pounds where n is a whole number.

   What is the smallest possible value of n?

3. **Rational Integer**
   For how many integers n is $\frac{n+3}{n-1}$ also an integer?

4. **Square LCM**
   The highest common factor of two positive integers m and n is 12, and their lowest common multiple is a square number.

   How many of the five numbers $\frac{n}{3}, \frac{m}{3}, \frac{n}{4}, \frac{m}{4}, mn$ are square numbers?

5. **Cancelling Fractions**
   Find a fraction $\frac{m}{n}$ $(m \neq n)$ such that all of the fractions

   $$\frac{m}{n}, \frac{m+1}{n+1}, \frac{m+2}{n+2}, \frac{m+3}{n+3}, \frac{m+4}{n+4}, \frac{m+5}{n+5}$$

   can be simplified by cancelling.

6. **Super Computer**
   Catherine's computer correctly calculates $\frac{66^{66}}{2}$.
   What is the units digit of the answer?

These problems are adapted from UKMT (ukmt.org.uk) and WMC (competition.ac) problems.