Number Operations and Calculation Methods

Age 11+ Level ★★★
Worksheet 1

1. **Last Digit**
   What is the last digit of $7^{2016} + 8^{2016}$?

2. **Sum of 1s**
   What are the last 5 digits of the sum $1 + 11 + 111 + 1111 + \ldots + \underbrace{111\ldots111}_{2016 \text{ digits}}$?

3. **Squares and Cubes**
   How many positive two-digit numbers are there whose square and cube both end in the same digit?

4. **Comparing Totals**
   The positive integer n is between 1 and 20.

   Milly adds up all the integers from 1 to n inclusive.
   Billy adds up all the integers from $n + 1$ to 20 inclusive.

   Their totals are the same. What is the value of n?

5. **Digit Deletion**
   Dominique wrote down the 1000-digit number 20082008…2008.
   She then erased some digits and was surprised to find that the remaining digits added up to 2008.

   What is largest number of digits that she could have erased?

6. **Sum One Special**
   Three different integers have a sum of 1 and a product of 36.

   What are they?

7. **Divisible by 55**
   Choose the digits a and b so that $227569ab$ is divisible by 55.
   There are two possible answers!

These problems are adapted from UKMT (ukmt.org.uk) and WMC (competition.ac) problems.