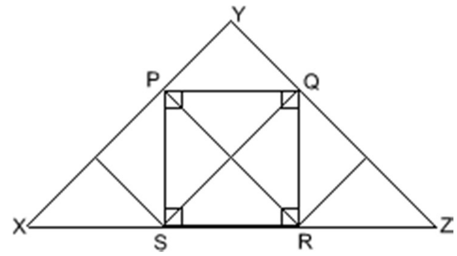
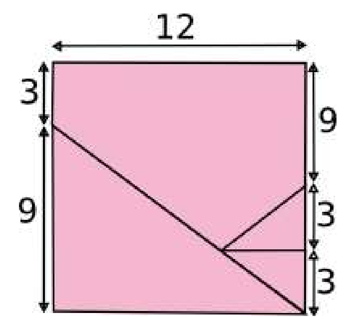
Stage 3 ★★
Mixed Selection 1 –Solutions

1. **Square in a triangle**
   The diagram shows that triangle XYZ may be divided into 9 congruent triangles. The square $PQRS$ is made up of 4 of these 9 triangles. Therefore, the area is $4:9$.



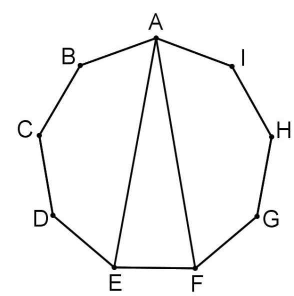
2. **Rectangle dissection**
   The square's perimeter is $12 \times 4 = 48$.

   You could also look at the area of the rectangle. This is $16 \times 9 = 144$. This must be the same as the square, so the square must have side lengths of 12. Therefore its perimeter is $12 \times 4 = 48$.



3. **Nonagon angle**
   The interior angle of a regular nine-sided polygon $= 180° - (360° \div 9) = 140°$.

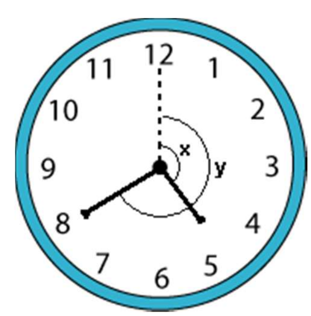
   Consider the pentagon $ABCDE$:
   $$\angle EAB = \frac{1}{2}(540° - 3 \times 140°) = 60°$$
   Similarly, $\angle FAI = 60°$ and hence $\angle FAE = 140° - (60° + 60°) = 20°$.



4. **Handy angles**
   Let x denote the angle between the hour hand and the vertical, and y the angle between the minute hand and the vertical.

   $\frac{360}{12} = 30$ so we calculate that $x = \left(4 + \frac{2}{3}\right) \times 30 = 140°$ and $y = 8 \times 30 = 240°$.

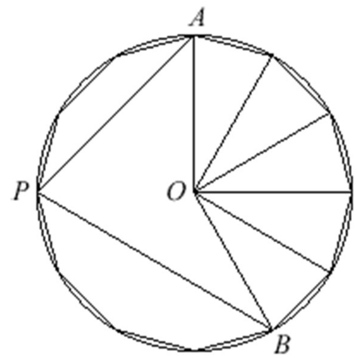
   Hence, the angle between the hands is $y - x = 240° - 140° = 100°$.



These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

5. Dodecagon angles

Each side of the dodecagon subtends an angle of $30°$ at the centre of the circumcircle of the figure (the circle which passes through all 12 of its vertices).



Since $\angle AOP = 90°$, $\angle OPA = 45°$. Since $\angle BOP = 120°$, $\angle OPB = 30°$. Therefore, $\angle APB = 45° + 30° = 75°$.

Alternatively, $\angle AOB = 150°$ and, as the angle subtended by an arc at the centre of a circle is twice the angle subtended by that arc at a point on the circumference, $\angle APB = 75°$.

6. Outside the boxes

At each vertex of the triangle, four angles meet which must add up to $360°$. Therefore, altogether the twelve angles add up to $3\,x\,360°$. Six of the angles are right-angles, and three of them are the internal angles of the triangle, which add up to $180°$.

So the sum of the remaining three angles must be

$$(3 \times 360°) - (6 \times 90°) - 180° = 360°.$$