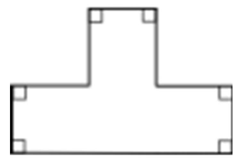
Stage 3 ★★
Mixed Selection 2 – Solutions

1. Right angled octagon

6 right angles can be achieved, as in the diagram below:



If there were seven right angles, these would be a total of $7 \times 90° = 630°$. The total interior angle of an octagon is $6 \times 180° = 1080°$, so the final angle would have to be $1080° - 630° = 450°$. The interior angle cannot be more than $360°$, so this cannot be achieved.

2. Fangs

Alternate angles BDF and DFG are equal, so lines BD and FG are parallel.
Therefore, $\angle BCA = \angle FGC = 80°$ (corresponding angles).

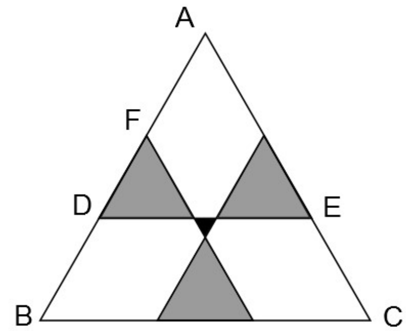
Consider triangle ABC: $x + 70 + 80 = 180$, so $x = 30°$.

3. Integral polygons

The greatest number of sides the polygon could have is 360.

As each interior angle of the polygon is a whole number of degrees, the same must apply to each exterior angle. The sum of the exterior angles of a polygon is $360°$ and so the greatest number of sides will be that of 360-sided polygon in which each interior angle is $179°$, thus making each exterior angle $1°$.

4. Radioactive triangle



As triangle ABC is equilateral, $\angle BAC = 60°$. Since the grey triangles are equilateral, $\angle ADE = 60°$, so the triangle ADE is equilateral.

The length of the side of this triangle is equal to the length of $DE = (5 + 2 + 5)\text{cm} = 12\text{cm}$. So $AF = AD - FD = (12 - 5)\text{cm} = 7\text{cm}$.

By a similar argument, we deduce that $BD = 7\text{cm}$, so the length of the side of the triangle $ABC = (7 + 5 + 7)\text{cm} = 19\text{cm}$.

5. Rhombus diagonal

Adjacent angles on a straight line add up to $180°$, so $\angle GJF = 180° - 111° = 69°$.
In triangle FGJ, $GJ = GF$ so $\angle GFJ = \angle GJF$.
Therefore, $\angle FGJ = (180 - 2 \times 69)° = 42°$.
Since $FGHI$ is a rhombus, $FG = FI$ and hence $\angle GIF = \angle FGI = 42°$.

Finally, from triangle FJI, $\angle JFI = (180 - 111 - 42)° = 27°$.

6. Inscribed hexagon

As the sum of the angles in a triangle is $180°$ and all four angles in a rectangle are $90°$, then sum of the two marked angles in the triangle $180° - 90° = 90°$.
Each interior angle of a regular hexagon is $120°$ and the sum of the angles in a quadrilateral is $360°$; hence the sum of the two marked angles in the quadrilateral is $360° - 90° - (360° - 120°) = 30°$.
Hence the sum of the four marked angles is $90° + 30° = 120°$.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)