

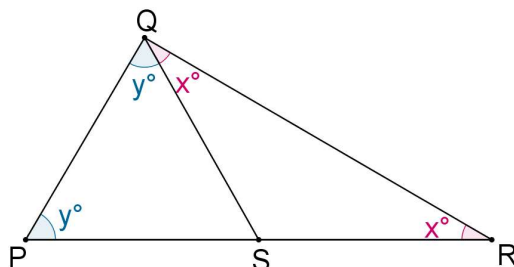
**Stage 3 ★★****Mixed Selection 3 – Solutions****1. Triangle split**

Since $SP = SQ$, the triangle PSQ is isosceles. Therefore, $\angle SPQ = \angle PQS$. We denote the measure of those angles by y .

Similarly, $\angle SQR = \angle QRS = x^\circ$.

Since the sum of the interior angles of PQR is 180° , $x + y + (x + y) = 180^\circ$,
so $2x + 2y = 180^\circ$.

Therefore, $\angle PQR = x^\circ + y^\circ = 90^\circ$.

**2. Hexapentagon**

Each interior angle of a regular pentagon is 108° , whilst each interior angle of a regular hexagon is 120° . The non-regular pentagon in the centre of the diagram contains two angles which are interior angles of the regular hexagon, two angles which are interior angles of the regular heptagon and a fifth angle, the one marked x .

So: $x + 2 \times 120 + 2 \times 108 = 5 \times 108 = 540$. Hence $x = 84$.

3. Extended parallelogram

Opposite angles of a parallelogram are equal, so $\angle QPS = 50^\circ$.

Therefore, $\angle QPT = 112^\circ$ and, as triangle QPT is isosceles, $\angle PQT = (180^\circ - 112^\circ)/2 = 34^\circ$.

As $PQRS$ is a parallelogram, $\angle PQR = 180^\circ - 50^\circ = 130^\circ$.

So, $\angle TQR = 130^\circ - 34^\circ = 96^\circ$.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)



Angles, Polygons and Geometrical Proof

4. Six minutes past eight

At 8 o'clock, the obtuse angle between the hands of the clock is 120° . In the following six minutes, the minute hand turns through an angle of 36° whilst the hour hand turns through an angle of 3° in the same direction (clockwise!). So the obtuse angle between the hands increases by 33° , to 153° .

5. U in a pentagon

Each interior angle of a regular pentagon is 108° , so $\angle SRQ = 108^\circ$.

As $SR = QR$, the triangle is isosceles with $\angle RQS = \angle RSQ = 36^\circ$.

Similarly, $\angle SRT = \angle STR = 36^\circ$. So $\angle SUR = (180 - 2 \times 36)^\circ = 108^\circ$.

From the symmetry of the figure,

$$\angle PUR = \angle PUS = (360^\circ - 108^\circ)/2 = 126^\circ.$$

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