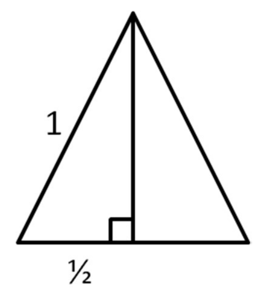
Stage 4 ★★
Mixed Selection 2 – Solutions

1. **Two right angles**
Triangles QPR and RPS are similar. So: $\frac{PR}{PS} = \frac{PQ}{PR}$. Hence $PR^2 = PQ \times PS = \frac{7}{3} \times \frac{48}{7} = 16$. So PR is 4 units long.

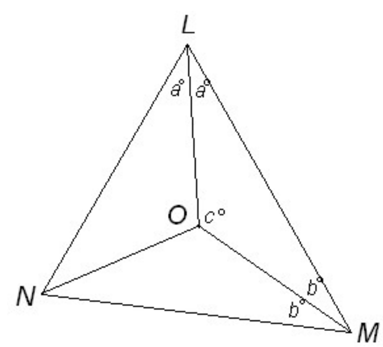
2. **Internal – external**
$\angle PSR = 90°$, as it is in a square.
$\angle USR = \angle TSP = 60°$, as they are in equilateral triangles.



Then, $\angle PSU = 90° - 60° = 30°$.
$\angle TSU = 60° + 30° = 90°$.

Thus, TSU is a right angled triangle, so using Pythagoras' theorem: $UT = \sqrt{1^2 + 1^2} = \sqrt{2}$.

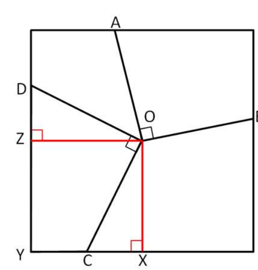
3. **Incentre angle**
Let $\angle OLM = \angle OLN = a°, \angle OML = \angle OMN = b°$ and $\angle LOM = c°$.



Angles in a triangle add up to 180°, so from $\Delta LMN, 2a° + 2b° + 68° = 180°$, which gives $2(a° + b°) = 112°$. In other words $a° + b° = 56°$.

Also, from $\Delta LOM, a° + b° + c° = 180°$, and so $c° = 180° - (a° + b°) = 180° - 56° = 124°$.

4. **Shaded square**
Consider connecting O to X and Z, both of which are the midpoints of the sides. Then, $OX = OZ$ as O is the centre of the square, and $\angle OXC = 90° = \angle OZD$.



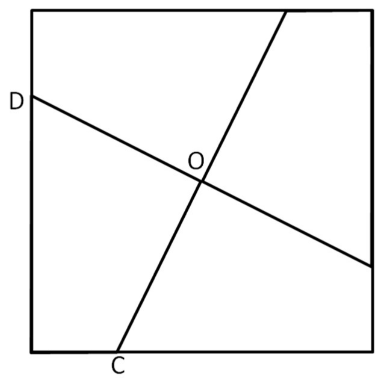
Then, $\angle DOC = 90° = \angle ZOX$, so:
$\angle DOZ = 90° - \angle ZOC = \angle COX$. This makes DOZ and COX congruent, as they have the same angles and one pair of corresponding sides the same length.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

But then, $CODY$ and $ZOXY$ have the same areas, as they both attach one of these triangles to $COZY$. This is clearly a quarter of the large square.

There are two of these unshaded areas, and both have area one quarter that of the square. This means half the square is shaded, so $12 \times 22 = 2\text{m}^2$.

Alternatively, extending the lines from D and C past O divides the square into four congruent pieces, since rotation by $90°$ takes each piece to the next. Therefore, each unshaded area is one quarter of the total area of the square.



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