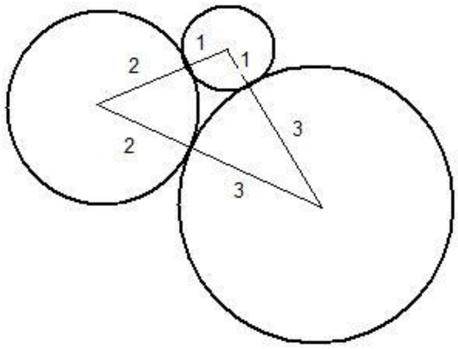
Stage 4 ★★
Mixed Selection 1 – Solutions

1. Circle time

The triangle that joins up the centres of the circles has sides of length 3 cm, 4 cm, 5 cm. Since there is a right-angled triangle with sides 3, 4, 5, and since if two triangles share the lengths of their sides they are congruent, this angle must be right-angled. Therefore the length of the longer arc of the circle C_1 is $\frac{3}{4}\times 2\pi\times 1=32\pi$ cm.

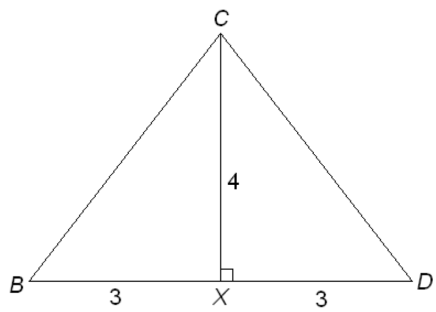


2. Unusual Polygon

The area of square *BDFG* is 6×6=36 square units.
So the total area of the three triangles *ABG*, *BCD* and *DEF* is also 36 square units. Since these triangles are congruent, each has an area of 12 square units.

The area of each triangle is $\frac{1}{2}\times \text{base}\times \text{height}$, and the base is 6 units and hence we have $\frac{1}{2}\times 6\times \text{height}=12$, so the height is 4 units.

Let *X* be the midpoint of *BD*. Then *CX* is perpendicular to the base *BD* (since *BCD* is an isosceles triangle).



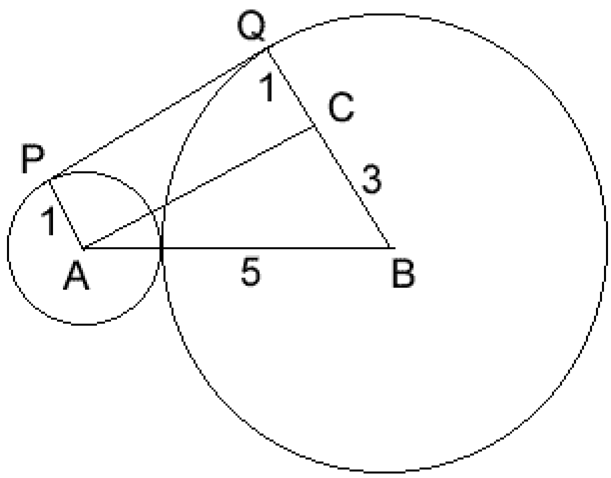
By Pythagoras' Theorem, $BC=\sqrt{3^2+4^2}=5$

Therefore the perimeter of *ABCDEFG* is

$$6\times 5+6=36$$

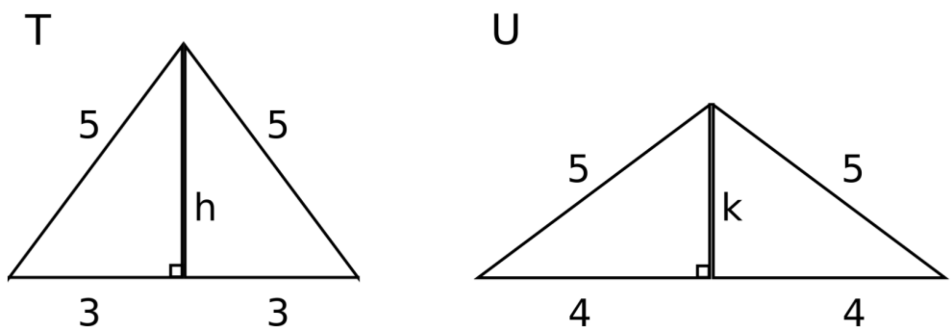
3. Common tangent

The diagram shows points A and B, the centres of the two circles, and C, on line BQ such that AC is parallel to PQ. Since ACQP is a rectangle, ACB is a right angled triangle. So By Pythagoras' Theorem, AC=4, which is also the length of PQ.



4. Triangular teaser

The diagram below shows isosceles triangles T and U. The perpendicular from the top vertex to the base divides an isosceles triangle into two congruent right-angled triangles as shown in both T and U. Evidently, by Pythagoras' Theorem, h=4 and k=3. So both triangles T and U consist of two 3, 4, 5 triangles and therefore have equal areas.



These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).