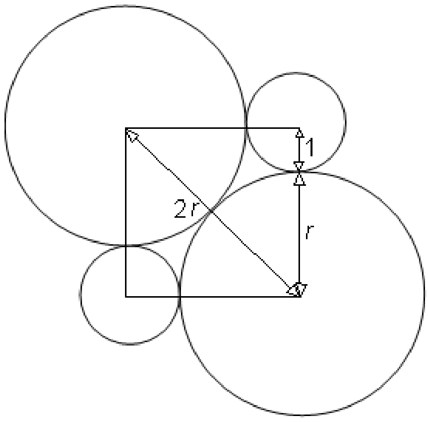
Stage 4 ★★
Mixed Selection 2 – Solutions

1. Centre square

Let r be the radius of each of the larger circles.
The sides of the square are equal to $r+1$, the sum of the two radii.
The diagonal of the square is $2r$.



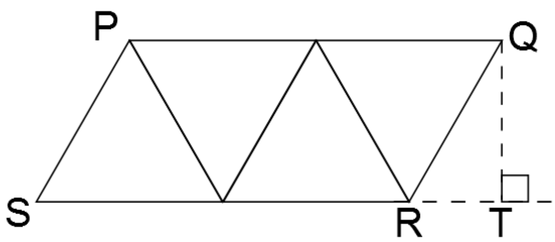
By Pythagoras, $(r+1)^2+(r+1)^2=(2r)^2$
Simplifying gives: $(r+1)^2=2r^2$
so $r+1=\sqrt{2}r$

Therefore $(\sqrt{2}-1)r=1$
Hence $r=\frac{1}{\sqrt{2}-1}=\sqrt{2}+1$

2. Crane arm

In the diagram, T is the foot of the perpendicular from Q to the extension of SR. All of the angles in the equilateral triangles are 60°, so ∠QRT is also 60°. Then ΔQRT is a right-angled triangle, so using trigonometry we can show that the lengths of RT and QT are $\frac{1}{2}$ and $3\sqrt{2}$ respectively.
Applying Pythagoras' Theorem to ΔQST,

$$SQ^2=ST^2+QT^2=5^2+\left(\sqrt{3}\right)^2=25+3=28$$



So the length of SQ is $\sqrt{28}=2\sqrt{7}$ units.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

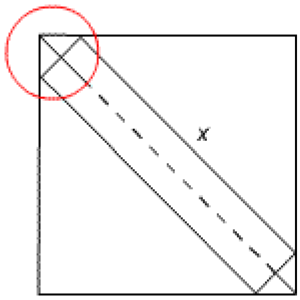
3. Walk the plank

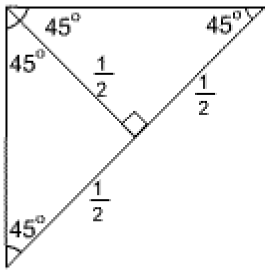
The figure shows the top left-hand corner of the complete diagram. Note the symmetry which leads to the three measurements of $\frac{1}{2}$. Thus the diagonal of the square can be divided into three portions of lengths:

$\frac{1}{2}, x$ and $\frac{1}{2}$ respectively.

The length of the diagonal is $\sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

So $x = 10\sqrt{2} - 1$.





4. Indigo interior

Let the centre of the circle be *O* and let *A* and *B* be corners of one of the shaded squares, as shown.

The circle has area π square units, so *OB* is 1 unit long.

Let the length of the side of each of the shaded squares be *x* units.
By Pythagoras, $OB^2 = OA^2 + AB^2$; that is $1^2 = (2x)^2 + x^2$.
So $5x^2 = 1$.
Now the total shaded area is $8x^2 = \frac{8}{5}$ square units.