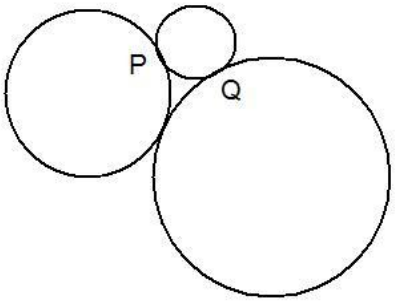
Stage 4 ★★
Mixed Selection 1

1. Circle time

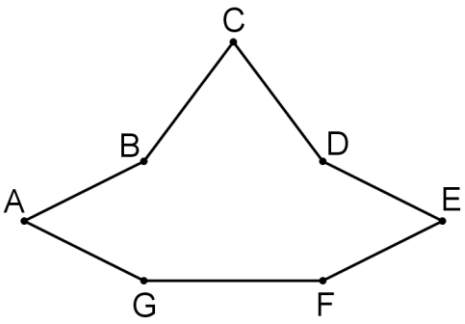
Three circles C_1, C_2 and C_3, of radii 1 cm, 2 cm and 3 cm respectively touch as shown. C_1 meets C_2 at P, and meets C_3 at Q.



What is the length in cm of the longer arc of circle C_1 between P and Q?

2. Unusual polygon

The diagram shows a polygon *ABCDEFG*, in which $FG=6$ and $GA=AB=BC=CD=DE=EF$.

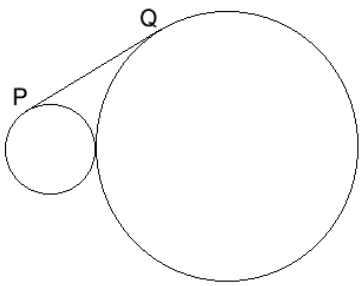


BDFG is a square, and the area of the whole polygon is exactly twice the area of *BDFG*.

What is the perimeter of the polygon?

3. Common tangent

Two circles with radii 1cm and 4cm touch. The point P is on the smaller circle, Q is on the larger circle, and PQ is a tangent to both circles.



What is the length of PQ?

4. Triangular teaser

The triangle *T* has sides of length 6, 5, 5. The triangle *U* has sides of length 8, 5, and 5.

What is the ratio area *T* : area *U*?

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).