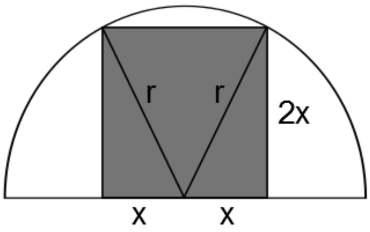
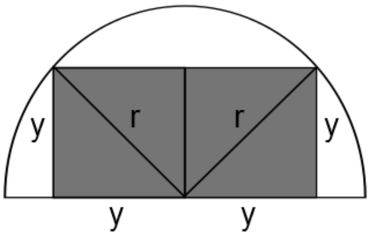
Stage 4 ★★★
Mixed Selection 1 – Solutions

1. One or two

Let the radius of each semicircle be r.
In the top diagram, let the side-length of the square be $2x$.
By Pythagoras' Theorem, $x^2 + (2x)^2 = r^2$ and so $5x^2 = r^2$.
So this shaded area is $(2x)^2 = 4x^2 = \frac{4r^2}{5}$.



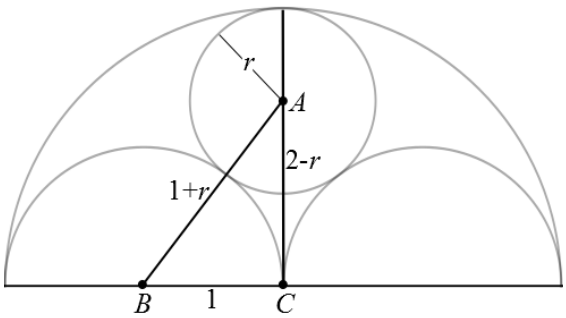
In the bottom diagram, let the side-length of each square be y. Then by Pythagoras' Theorem, $y^2 + y^2 = r^2$ and so $2y^2 = r^2$.
So this shaded area is r^2.



Therefore the ratio of the two shaded areas is 4:5.

2. Salt's Mill

Let *A* be the centre of the circle, *B* be the centre of the left-hand semicircle and *C* be the centre of the large semicircle. Then we can mark their lengths below:



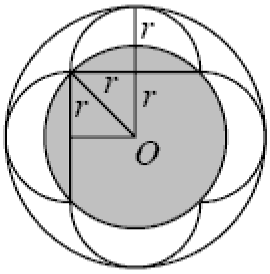
By Pythagoras' Theorem:
$(1+r)^2=1^2+(2-r)^2$
$1+2r+r^2=1+4-4r+r^2$
$6r=4$

So the radius is 2/3 of a metre.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

3. Oh so circular

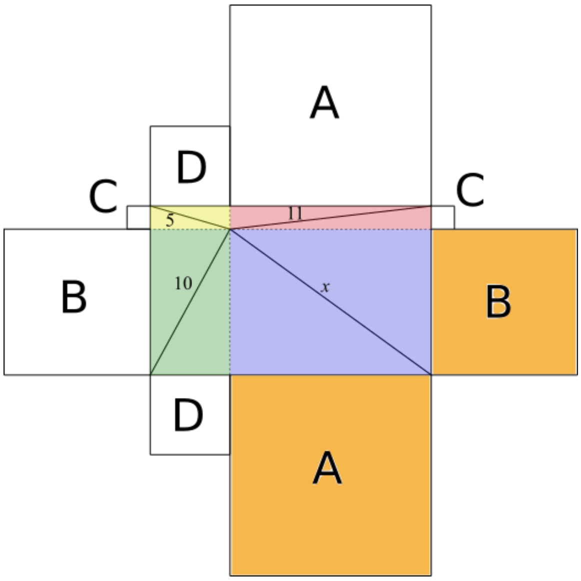
From the symmetry of the figure, the two circles must be concentric. Let their centre be O. Let the radius of the semicircles be r. Then the radius of the outer circle is $2r$ and, by Pythagoras' Theorem, the radius of the inner shaded circle is $\sqrt{r^2+r^2}$, that is $\sqrt{2}r$.



So the radii of the two circles are in the ratio $\sqrt{2}:2$, and hence the ratio of their areas is $2:4=1:2$.

4. Distance to the corner

In the diagram below, the courtyard has been split into 4 rectangles whose corners are at the well. The squares drawn on are for the diagrammatic representation of Pythagoras' theorem, so that the area of shaded square A added to the area of shaded square B is equal to x^2, because x is the hypotenuse of the triangle in the blue rectangle.



The square of the hypotenuse in the red rectangle is A+C, and the square of the green rectangle is B+D. Adding these together gives A+B+C+D, which is too much by C+D. But the square of the hypotenuse in the yellow rectangle is C+D.

So A+B $=11^2+10^2-5^2=196$.

Then $x^2=196$, so $x=14$.

A fuller solution is available at: https://nrich.maths.org/12899/solution

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