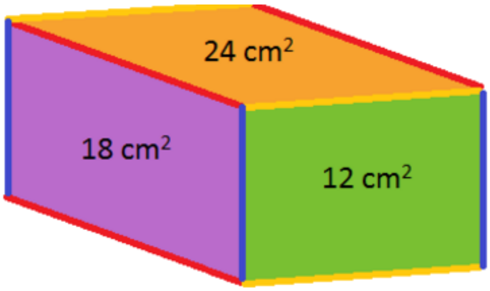
Stage 3 ★
Mixed Selection 3 - Solutions

1. Cuboid faces



From the above diagram we can see that:

- the length of the yellow edge multiplied by the length of the blue edge must be 12.

-the length of the red edge multiplied by the length of the blue edge must be 18.

-the length of the yellow edge multiplied by the length of the red edge must be 24.

We can start by thinking of two numbers that multiply to get 12, and then choose the right number to make the purple face have an area of 18 square centimetres. Then we can check whether those numbers give us the correct area for the orange face.

If the yellow edge is 6 cm long and the blue edge is 2 cm long, then the red edge must be 9 cm long, because $2\times9 = 18$. However, then the orange face would have area $6\times9 = 54$ square centimetres, which is not right.

If the yellow edge is 2 cm long and the blue edge is 6 cm long, then the red edge must be 3 cm long, because $6\times3 = 18$. However, then the orange face would have area $2\times3 = 6$ square centimetres, which is not right.

If the yellow edge is 4 cm long and the blue edge is 3 cm long, then the red edge must be 6 cm long, because $3\times6 = 18$. Then the orange face would have area $4\times6 = 24$ square centimetres, which is right.

So the edges of the cuboid are 3 cm, 4 cm and 6 cm long, which means the volume of the cuboid is $3\times4\times6 = 72\ cm^3$.

A fuller solution is available at: http://nrich.maths.org/12466/solution

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.

2. Strawberries and peas

The length of the pea bed was increased by 3m,so the length of the strawberry patch was reduced by 3m. This reduced the area by 15m^2. Therefore, the rectangle that was transferred had length 3m and area 15m^2. The width of the rectangle is therefore $15 \div 3 = 5$, so the garden is 5m wide.

The new pea bed is a square. As it is 5m wide, it is also 5m long. This is after it had a side increased by 3m, so originally it was $5\text{m} \times 2\text{m}$. The area of this is $5 \times 2 = 10\text{m}^2$.

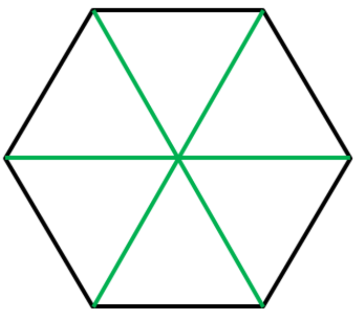
3. Giant Rubik's cube

The outside cubes enclose a 10 x 10 x 10 cube, so the number of visible cubes is the total cubes minus these. This is $12^2 - 10^3 = 728$.

A fuller solution is available at: https://nrich.maths.org/12840/solution

4. Star in a hexagon

The diagram on the right demonstrates how each of the small hexagons can be split into six of the small triangles. The shaded area is therefore the equivalent of 12 of the small triangles. The whole large hexagon is the equivalent of
$7 \times 6 + 12 = 54$ small triangles.



Therefore the shaded area is $\frac{12}{54} = \frac{2}{9}$ of the whole hexagon.

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.