

**Stage 3 ★★****Mixed Selection 2 – Solutions****1. Intersecting squares**

Each of the overlapping areas contributes to the area of exactly two squares. So the total area of the three squares is equal to the area of the non-overlapping parts of the squares plus twice the total of the three overlapping areas,

$$\text{i.e. } (117 + 2(2 + 5 + 8))\text{cm}^2 = (117 + 30)\text{cm}^2 = 147\text{cm}^2.$$

So the area of each square is $(147 \div 3)\text{cm}^2 = 49\text{cm}^2$. Therefore the length of the side of each square is 7cm.

2. Cubic masterpiece

Let the smaller cubes have side length 1unit. So the original cube had side of length 3unit and, as a cube has six faces, it had a surface area of $6 \times (3\text{units} \times 3\text{units}) = 54\text{units}^2$, all of which was painted blue.

The total surface area of the 27 small cubes is $27 \times 6\text{units}^2 = 162\text{units}^2$.

So the required fraction is $\frac{54\text{units}^2}{162\text{units}^2} = \frac{1}{3}$.

3. Cubes on a cube

The shape consists of 7 cubes, and has a total volume of 875cm^3 . Each cube therefore has volume $875\text{cm}^3 \div 7 = 125\text{cm}^3$.

Therefore, the side length of the cube is $\sqrt[3]{125\text{cm}^3} = 5\text{cm}$, so each face has area $(5\text{cm})^2 = 25\text{cm}^2$.

Each of the outer cubes has five of its faces showing, so there are $6 \times 5 = 30$ faces showing altogether. These have a total area of $30 \times 25\text{cm}^2 = 750\text{cm}^2$.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

**4. Sideways ratio**

Let the sides of the rectangle, in cm, be $4x$ and $5x$.

Then the area of the square is $4x \times 5x = 20x^2 \text{ cm}^2$. So $20x^2 = 125$, that is $x^2 = 6.25$. Therefore $x = \pm 2.5$, but x cannot be negative so $x = 2.5$ and so the sides of the rectangle are 10cm and 12.5cm.

Hence the rectangle has perimeter 45cm.

5. Exactly three-quarters

The area of the rectangle is 48 cm^2 , so the unshaded area is 12 cm^2 .

Therefore taking the two unshaded triangles together:

$$\left(\frac{1}{2} \times x \times 2\right) + \left(\frac{1}{2} \times (6 - x) \times 8\right) = 12$$

which means that $x + 24 - 4x = 12$, so $x = 4$.