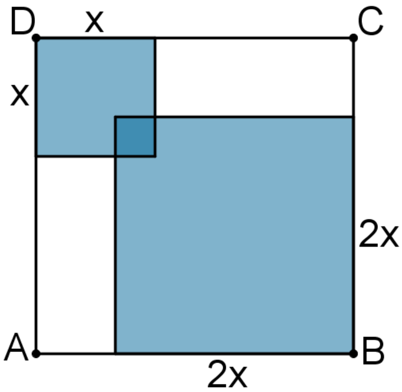
Stage 3 ★★★
Mixed Selection 1 – Solutions

1. Four square



The large square has area $196 = 14^2$, so it has side-length 14. The ratio of the areas of the inner squares is $4:1$, so the ratio of their side-lengths is $2:1$.

Let the side-length of the larger inner square be $2x$, so that of the smaller is x. The figure is symmetric about the diagonal AC and so the overlap of the two inner squares is also a square which therefore has side-length 1.

Thus the vertical height can be written as $x + 2x - 1$.
Hence, $3x - 1 = 14$ and so $x = 5$

Therefore the small shaded square has an area of $5 \times 5 = 25$ and the larger shaded square has an area of $10 \times 10 = 100$

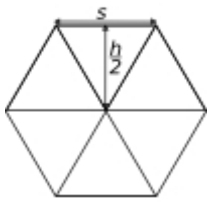
Therefore the total shaded area is $25 + 100 - 1 = 124$

Note also that the two unshaded rectangles have side-lengths $14 - 2x = 4$ and $14 - x = 9$

So the total unshaded area is $36 \times 2 = 72$ and therefore the total shaded area is $196 - 72 = 124$

2. Triangle in a hexagon

Call the length of one side of the hexagon s and the height of the hexagon h. So the area of the shaded triangle is $\frac{1}{2} \times s \times h$.

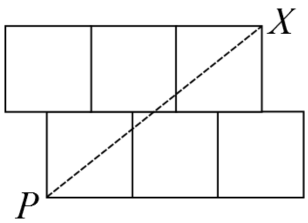


Now divide the hexagon into 6 equilateral triangles:

so the area of the hexagon is $6 \times 12 \times s \times \frac{h}{2} = 3 \times \frac{1}{2} \times s \times h$, or $3 \times \text{Shaded area}$. So the shaded area is 13 of the area of the whole hexagon.

3. Pile driver

The small square on top will be in the upper half of the divided figure. Now consider the figure formed by moving this square to become an extra square on the left of the second row, as shown.

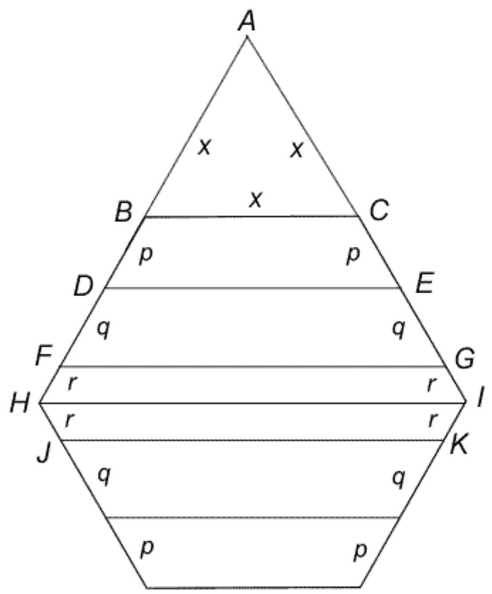


It may now be seen from the rotational symmetry of the figure that the line PX splits the new figure in half- with that small square in the upper half.

Therefore the line which halves the area of the original figure cuts the edge XY at X.

4. Hexagon slices

Each exterior angle of a regular hexagon is $\frac{360°}{6} = 60°$, so when sides HB and IC are extended to meet at A, an equilateral



triangle, ABC is created.

Let the sides of this triangle be of length x. As BC, DE and FG are all parallel, triangles ABC, ADE and AFG are all equilateral.
So, $DE = DA = p + x$; and $FG = FA = q + p + x$.

The perimeter of trapezium
$BCED = x + p + x + 2p = 2x + 3p$;

The perimeter of trapezium

$$DEGF = (p + x) + (q + p + x) + 2q$$
$$= 2x + 2p + 3q$$

The perimeter of hexagon
$FGIKJH = 2((q + p + x) + 2r) = 2x + 2p + 2q + 4r$.

So, $2x + 3p = 2x + 2p + 3q$; hence $p = 3q$.
Also $2x + 2p + 3q = 2x + 2p + 2q + 4r$; hence $q = 4r$.
So, $p : q : r = 12r : 4r : r = 12 : 4 : 1$.