Stage 4 ★★
Mixed Selection 1 - Solutions

1. Circled corners

The three angles of the triangle add to $180°$, so the combined area of the three sectors of the circles that are inside the triangle add up to half a circle with area:

$\frac{1}{2} \times \pi \times 2^2 = \frac{4\pi}{2} = 2\pi$.

So the grey area is $(80 - 2\pi)cm^2$.

2. Rolling inside

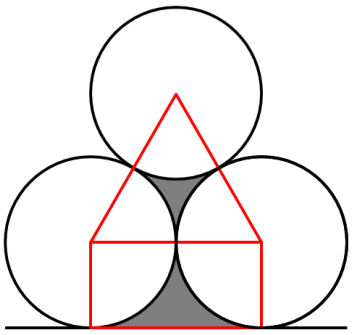
The circumference of the circle is 2π. This is the distance its centre moves each time the circle rolls for one revolution. When the circle moves from one corner to an adjacent corner, its centre moves a distance 2, so the circle makes $1/\pi$ revolutions. As it needs to do this four times before the circle returns to its original position, the number of revolutions is $4/\pi$.

3. Sticky tape

The cross section of the $20m$ tape has area $\pi(4^2 - 3^2)cm^2 = 7\pi\ cm^2$. Therefore, the $80m$ tape should have a cross-section area $28\pi\ cm^2$. Hence, the outer radius of the $80m$ roll will be approximately $\sqrt{37}cm$.

4. Wood pile perimeter

The centres of the three circles form an equilateral triangle, so each of the sectors marked is $1/6$ of a circle. The shape below is a rectangle, so the sectors are $1/4$ of a circle. The curved portion of the perimeters is therefore the same as each of the circles: $2\pi \times 5 = 10\pi cm$.



The straight part is the same length as two radii, so is $10cm$ long.

The perimeter of the shaded shapes is therefore $(10 + 10\pi)cm$.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)