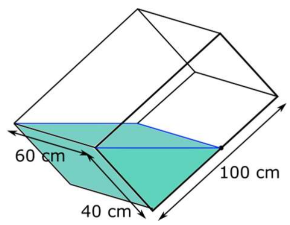
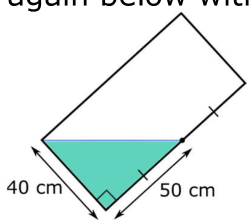
Stage 3 ★★
Mixed Selection 4 - Solutions

1. **Tilted tank**
The water in the tank is in the shape of a triangular prism, as shown, so its volume can be found using length × area of cross-section.

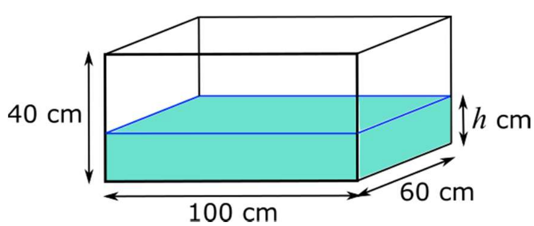


The cross-section is the triangle on the original diagram, shown again below with the sides labelled.



Because the two sides which are labelled are perpendicular, they can be considered to be the base and height of the triangle, so its area is $\frac{1}{2} \times 40 \times 50 = 1000\ cm^2$. So the volume of water in the tank is $60 \times 1000 = 60000\ cm^3$.

When the tank is resting on its base, as shown below, the volume of water in the tank is given by $100 \times 60h = 6000h\ cm^3$.

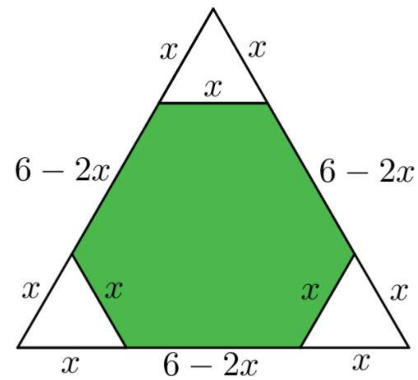


But the volume of water in the tank must also still be 60000 cm3, so $6000h = 60000$. So h must be 10.
So the height of the water is 10 cm.
A fuller solution is available at: http://nrich.maths.org/12782/solution

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.

2. Corner cut

Suppose the small equilateral triangles have sides of length x cm. Then, each of these has perimeter $3x$, and there are three of these, so the total perimeter is $3 \times 3x = 9x$.



The hexagon has three sides where the triangles have been cut from, each of length x. The other sides are the sides of the larger triangle, less two sides of the smaller triangles. They, therefore, have length $6 - 2x$. There are three of each of these, so the perimeter of the hexagon is $3x + 3(6 - 2x)$.

Since the hexagon has a perimeter equal to the sum of the three small triangles, we get: $9x = 3x + 3(6 - 2x)$
Then, expanding the bracket gives: $9x = 3x + 18 - 6x$.
Collecting like terms: $9x = 18 - 3x$.
Adding $3x$ to each side: $12x = 18$.
Dividing by 6: $x = 1.5$.

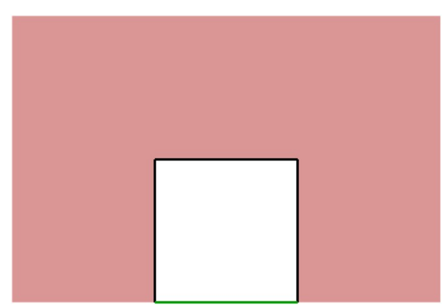
Therefore, the small equilateral triangles have sides of length 1.5cm.

3. Leaning over

The triangle HIJ has a base (HI) of 4cm, the same as the side of the square.
The area of the square is $4 \times 4 = 16\text{cm}^2$, so this is also the area of the triangle.
If the perpendicular height of HIJ is h, then $12 \times 4 \times h = 16$. This means $2h = 16$, so $h = 8\text{cm}$.
J is therefore 8cm above the top of the square, which is 4cm above the base, making a total of 12cm.

4. Christmas cut out

The original rectangle had a perimeter of $2 \times 30 + 2 \times 40 = 140cm$.
Each of the new squares that is cut out adds three sides of 5cm each (shown in black on the diagram), and removes one portion of length 5cm (shown in green).
This means the overall effect of cutting out each square is to increase the perimeter by 10cm.
Since are ten squares, the overall increase is $10 \times 10cm = 100cm$.
Therefore the final perimeter is $140cm + 100cm = 240cm$.



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