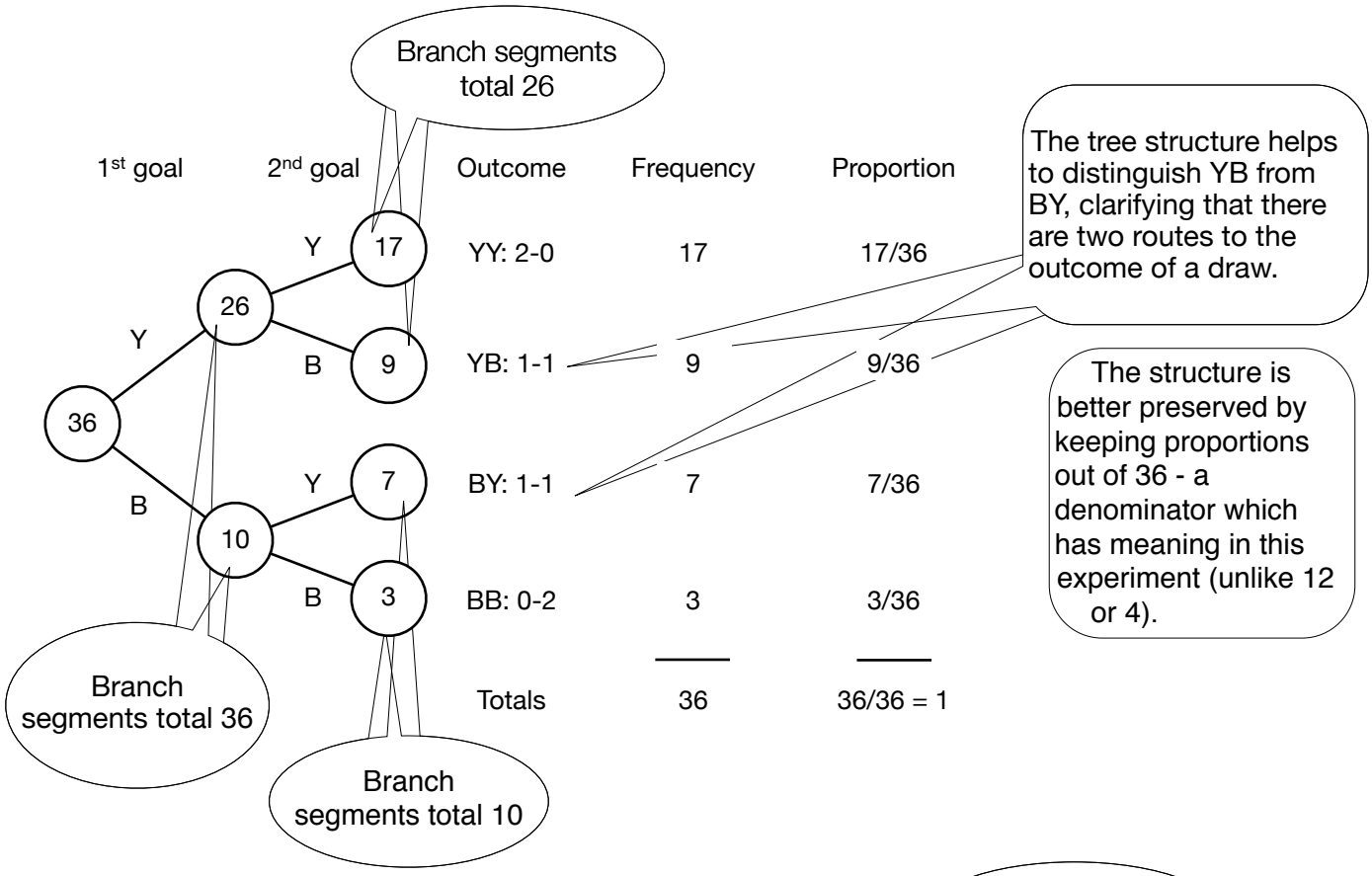
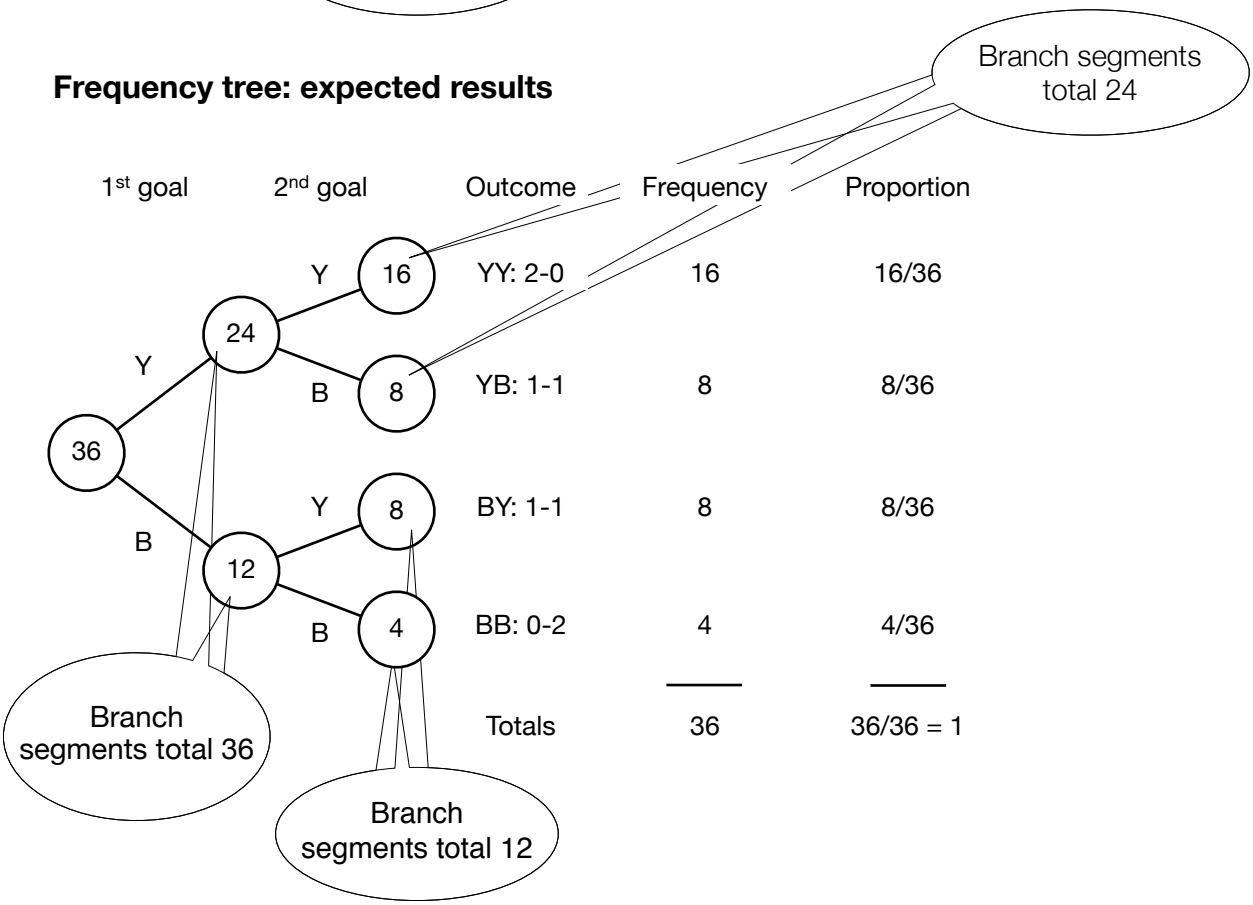
Representations: Which Team Will Win?

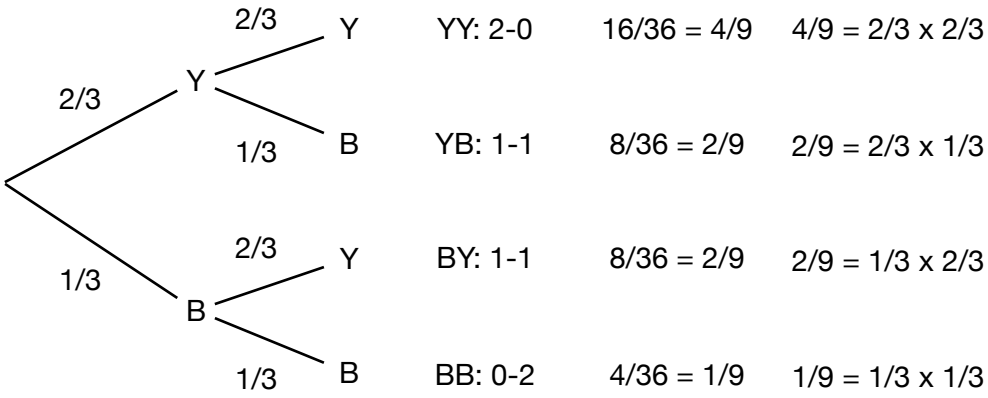
Frequency tree: sample experimental results



Frequency tree: expected results



Probability Tree



On a frequency tree, the total for each pair of branch segments is equal to the previous value, from which they fork. On a probability tree the sum of the probabilities on each pair of branch segments is 1. In this problem, the probabilities are the same for the 1st goal and the 2nd goal, indicating that the two goals are independent of each other, ie. scoring the first goal does not affect the probability of scoring the second goal.

The progression from empirical frequency tree to expected frequency tree to probability tree

- The trees are structurally the same, in that the first pair of branches gives the two options for the first goal, and the second pairs of branches give the options for the second goal.
- The progression is:
  - observed (empirical) frequency tree: a description of a past series of experiments, based on empirical observation
  - expected frequency tree: an expectation of a future series of experiments, based on theoretical reasoning
  - probability tree: a representation of a single future experiment, based on theoretical reasoning

From expected proportions to probability calculations

The expected proportion of the 36 trials is equivalent to the probability that a particular outcome occurs. Starting from this as the answer enables students to see how the probability calculation (multiplying along the branches) mirrors the process by which they obtained the expected frequency of each outcome:

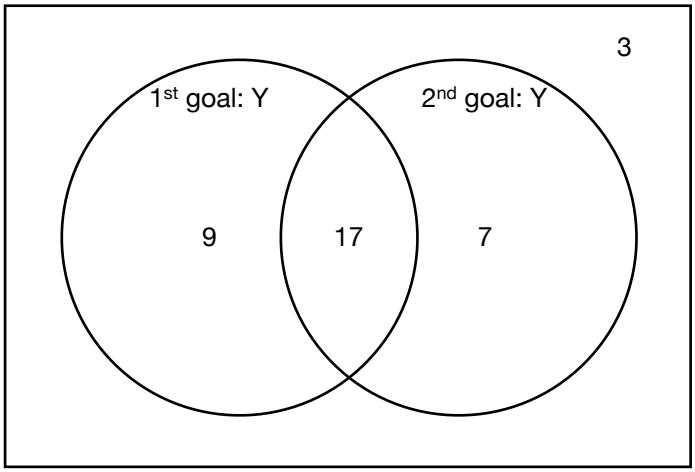
> On average, the Yetis will score 2/3 of the first goals (2/3 of 36 is 24), so the probability that they score the first goal is 2/3.
>
> On average, they will also score 2/3 of the second goals (2/3 of 24 is 16), so the probability that they score the second goal also is 2/3 of 2/3, or 2/3 x 2/3 = 4/9, which is equivalent to the expected proportions of wins for them of 16/36.

2-way Tables and Venn Diagrams

Sample experimental results

		2nd goal		
		Y	B	1st goal totals
1st goal	Y	17	9	26
	B	7	3	10
	2nd goal totals	24	12	36

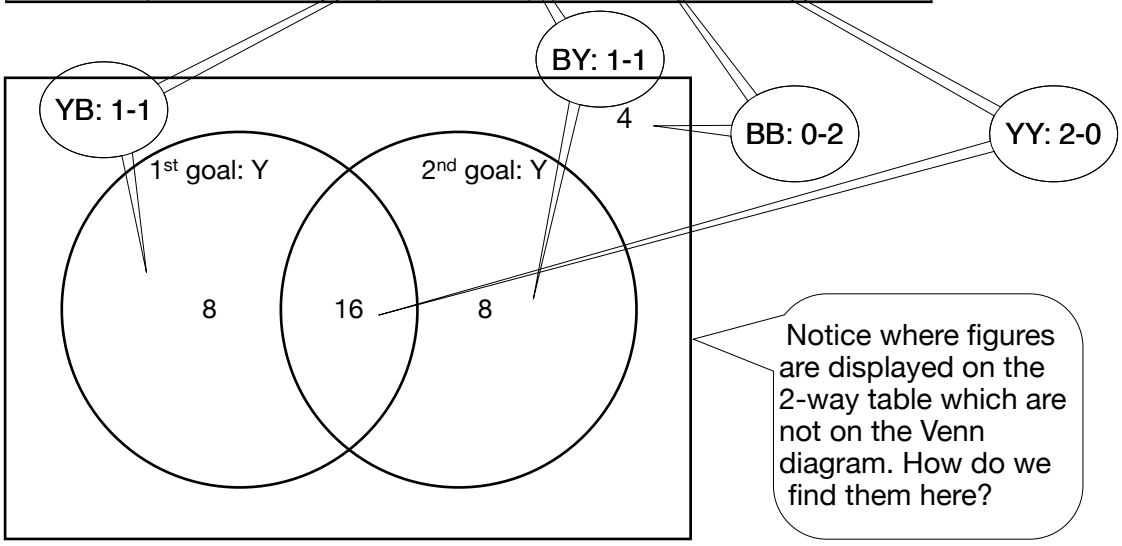
Notice where figures from the tree are displayed on the 2-way table. Which are not on the tree? How do we find them there?



It can be tricky to label the circles correctly on a Venn diagram. They correspond to the two stages of the tree diagram, not to the two teams. So who scores first, and who scores second? The labels could refer to B rather than Y, therefore.

Expected results

		2nd goal		
		Y	B	1st goal totals
1st goal	Y	16	8	24
	B	8	4	12
	2nd goal totals	24	12	36



Notice where figures are displayed on the 2-way table which are not on the Venn diagram. How do we find them here?