Frequency tree: expected results for Lucy's suggestion

No possibility of a girl for the first choice
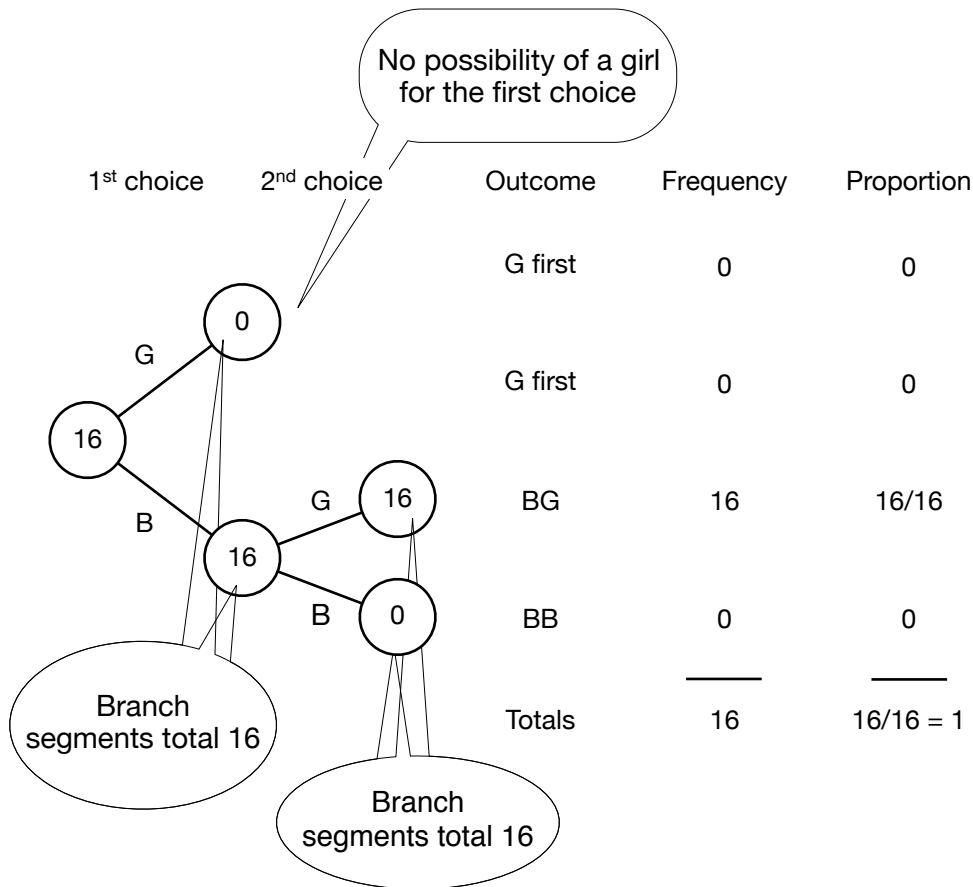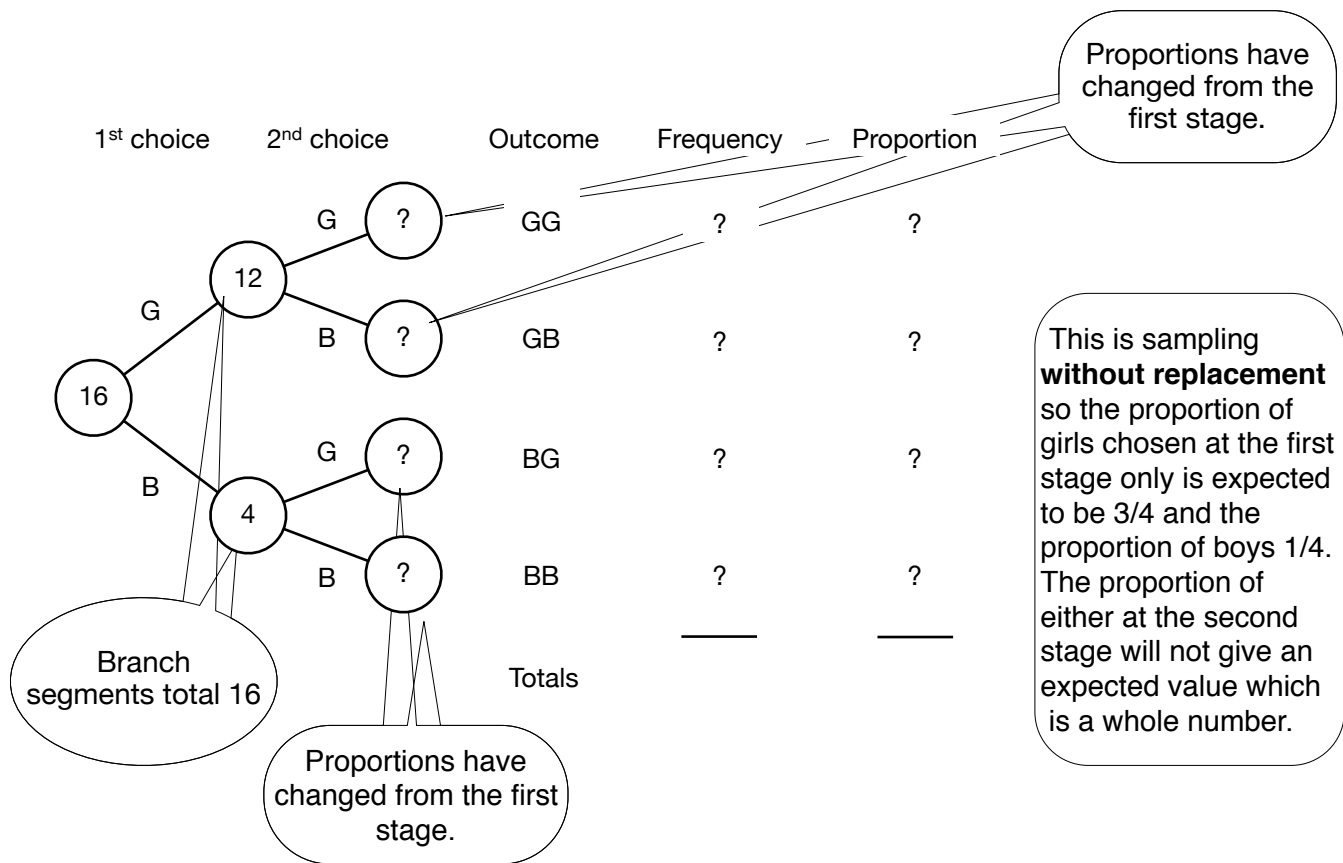
There is only one possible outcome for this scenario - first a boy is chosen, then a girl, so that will be the result in all 16 trials.

1st choice	2nd choice	Outcome	Frequency	Proportion
		G first	0	0
		G first	0	0
		BG	16	16/16
		BB	0	0
		Totals	16	16/16 = 1

Branch segments total 16

Branch segments total 16

Frequency tree: expected results for Ingrid's suggestion

Branch segments total 12

1st choice	2nd choice	Outcome	Frequency	Proportion
		GG	11	11/36
		GB	1	1/36
		BG	3	3/36
		BB	21	21/36
		Totals	36	36/36 = 1
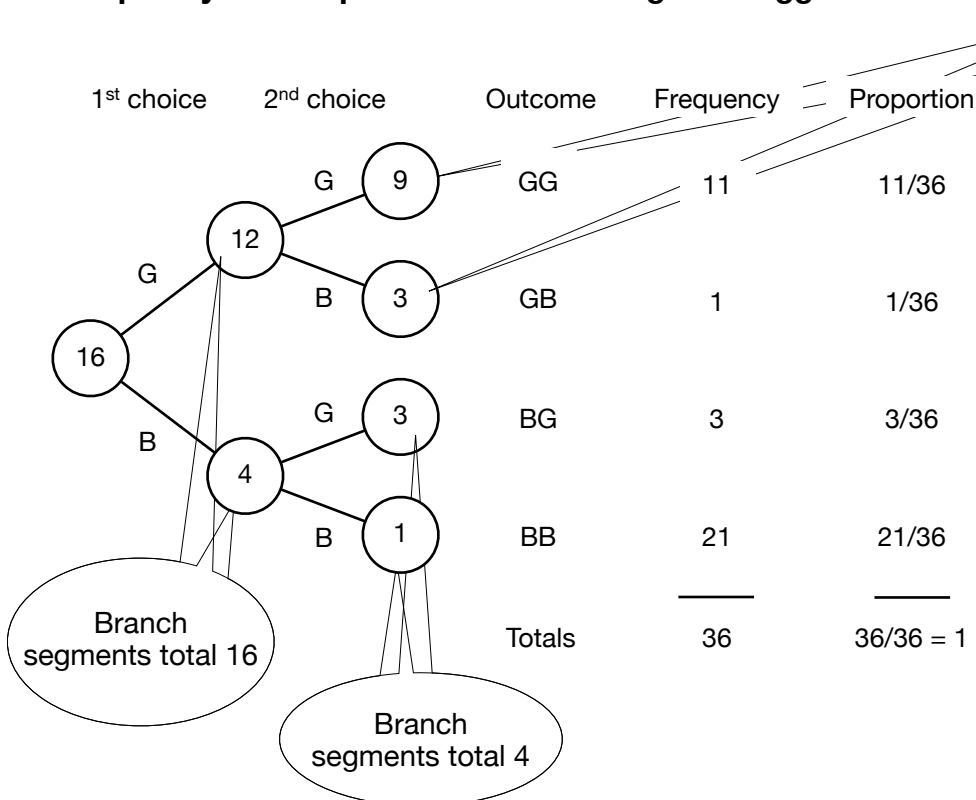
This is sampling **with replacement** so the proportion of girls chosen at both stages is expected to be 3/4 and the proportion of boys 1/4.

Branch segments total 16

Branch segments total 4

https://nrich.maths.org/9843

Frequency tree: expected results for Paolo's suggestion

1st choice   2nd choice   Outcome   Frequency — Proportion

Proportions have changed from the first stage.

G — ? — GG — ? — ?

12

G

B — ? — GB — ? — ?

16

G — ? — BG — ? — ?

B

4

B — ? — BB — ? — ?

Totals

Branch segments total 16

Proportions have changed from the first stage.

This is sampling **without replacement** so the proportion of girls chosen at the first stage only is expected to be 3/4 and the proportion of boys 1/4. The proportion of either at the second stage will not give an expected value which is a whole number.

Paolo's suggestion, which is sampling **without** replacement, means that the chance of a girl or boy being chosen changes from the first choice to the second, whereas Ingrid's suggestion, which is sampling **with** replacement, means that the chance remains the same for the second choice. Sampling **with** replacement also means that the same person can be chosen both times, which is not possible with sampling **without** replacement.
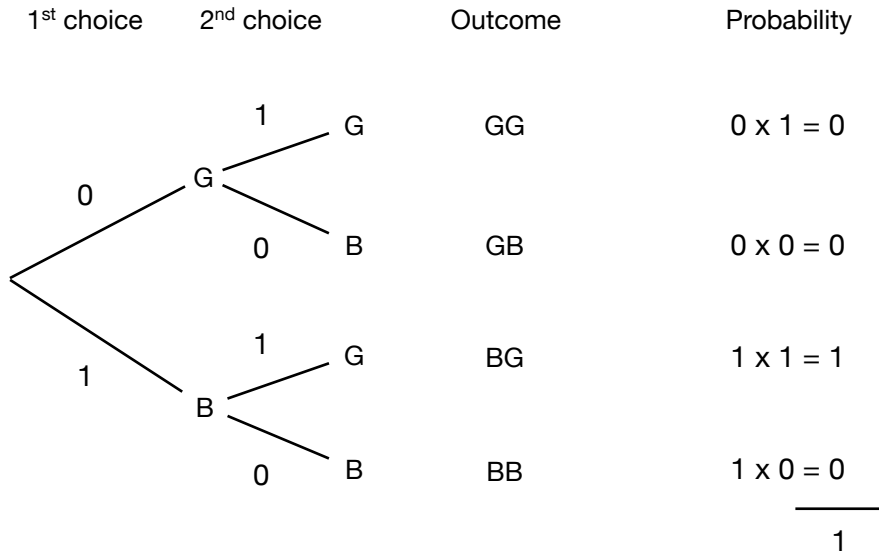
The suggested number of trials for the experiments is 16, because this gives expected results in whole numbers for both Lucy's and Ingrid's suggestions. Analysis of the expected results is more difficult once we look at Paolo's suggestion. This is the point at which we really need to move from expected results, expressed as proportions of the number of trials, to probabilities, which are normalised for a total probability of 1, since it makes no sense to talk of numbers of people which are not integers.

There are similar problems in analysing both Ingrid's and Paolo's suggestions when it comes to considering what we expect to happen for Alesha and Jack.
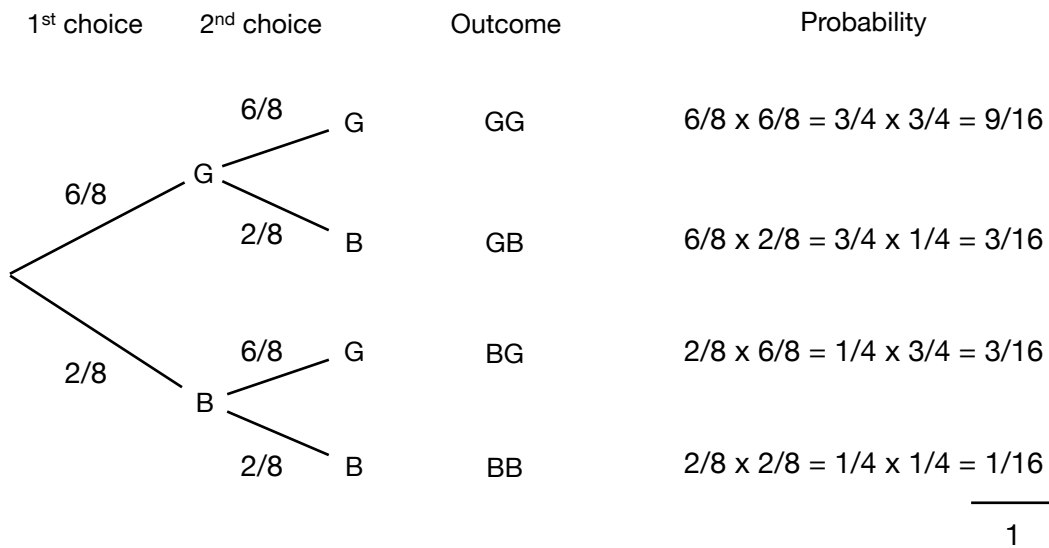
It is useful for students to do the experiments, and compare results for each suggestion, then to do the frequency trees as far as they make sense. This sets the scene for moving from frequency trees to probability trees, using the multiplication rule which should be familiar by this stage.

Trying to do the frequency tree for Paolo's suggestion provides an opportunity for discussion of how it differs from Ingrid's suggestion, and the problem this creates. It also ensures that students have fully recognised the change in the number of girls and boys available to be chosen when it comes to the second choice of a person.
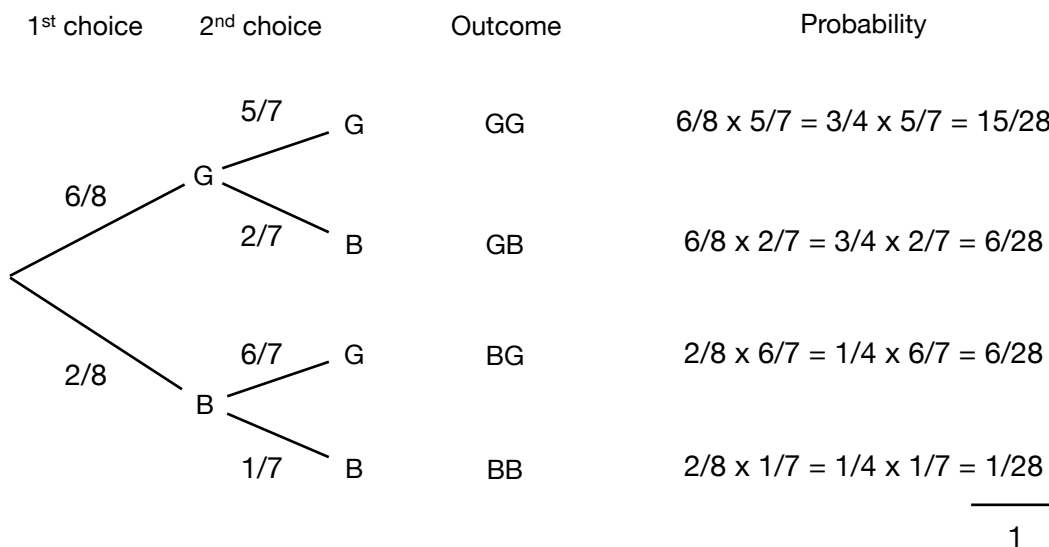
Probability Tree: Lucy's suggestion

1st choice	2nd choice	Outcome	Probability

G	1 → G	GG	0 x 1 = 0
0	0 → B	GB	0 x 0 = 0
1 → B	1 → G	BG	1 x 1 = 1
	0 → B	BB	1 x 0 = 0

$$\overline{\hspace{2cm}}$$
1

Probability Tree: Ingrid's suggestion

1st choice	2nd choice	Outcome	Probability

G	6/8 → G	GG	6/8 x 6/8 = 3/4 x 3/4 = 9/16
6/8	2/8 → B	GB	6/8 x 2/8 = 3/4 x 1/4 = 3/16
2/8 → B	6/8 → G	BG	2/8 x 6/8 = 1/4 x 3/4 = 3/16
	2/8 → B	BB	2/8 x 2/8 = 1/4 x 1/4 = 1/16

$$\overline{\hspace{2cm}}$$
1

Probability Tree: Paolo's suggestion

1st choice	2nd choice	Outcome	Probability

G	5/7 → G	GG	6/8 x 5/7 = 3/4 x 5/7 = 15/28
6/8	2/7 → B	GB	6/8 x 2/7 = 3/4 x 2/7 = 6/28
2/8 → B	6/7 → G	BG	2/8 x 6/7 = 1/4 x 6/7 = 6/28
	1/7 → B	BB	2/8 x 1/7 = 1/4 x 1/7 = 1/28

$$\overline{\hspace{2cm}}$$
1

Probability Trees for Alesha and Jack: Lucy's suggestion

1st choice	2nd choice	Alesha?	Probability
	1/6 A	Yes (x2)	0
0 A			
	5/6 A´	Yes	0
	1/6 A	Yes	1/6
1 A´			
	5/6 A´	No	5/6
			1

P(Alesha) = 1/6

1st choice	2nd choice	Jack?	Probability
	0 J	Yes (x2)	0
1/2 J			
	1 J´	Yes	1/2
	0 J	Yes	0
1/2 J´			
	1 J´	No	1/2
			1

P(Jack) = 1/2

Probability Trees for Alesha and Jack: Ingrid's suggestion

1st choice	2nd choice	Alesha?	Probability
	1/8 A	Yes (x2)	1/64
1/8 A			
	7/8 A´	Yes	7/64
	1/8 A	Yes	7/64
7/8 A´			
	7/8 A´	No	49/64
			1

P(Alesha) = 1/64 + 7/64 + 7/64 = 15/64

1st choice	2nd choice	Jack?	Probability
	1/8 J	Yes (x2)	1/64
1/8 J			
	7/8 J´	Yes	7/64
	1/8 J	Yes	7/64
7/8 J´			
	7/8 J´	No	49/64
			1

P(Jack) = 1/64 + 7/64 + 7/64 = 15/64

Probability Trees for Alesha and Jack: Paolo's suggestion

1st choice	2nd choice	Alesha?	Probability
	0 A	Yes (x2)	0
1/8 A			
	1 A´	Yes	1/8
	1/7 A	Yes	1/8
7/8 A´			
	6/7 A´	No	6/8
			1

P(Alesha) = 1/8 + 1/8 = 1/4

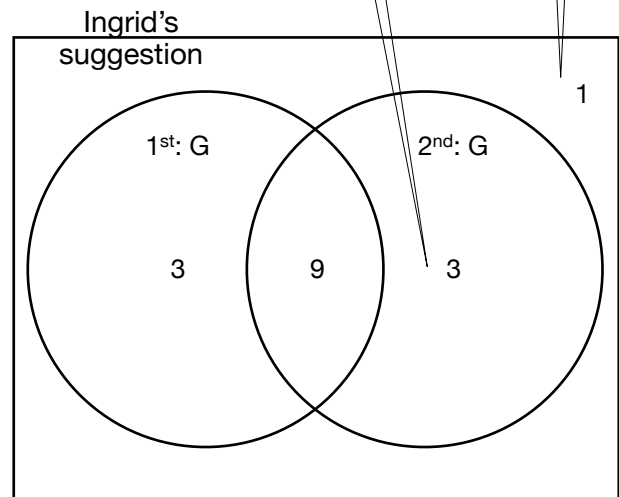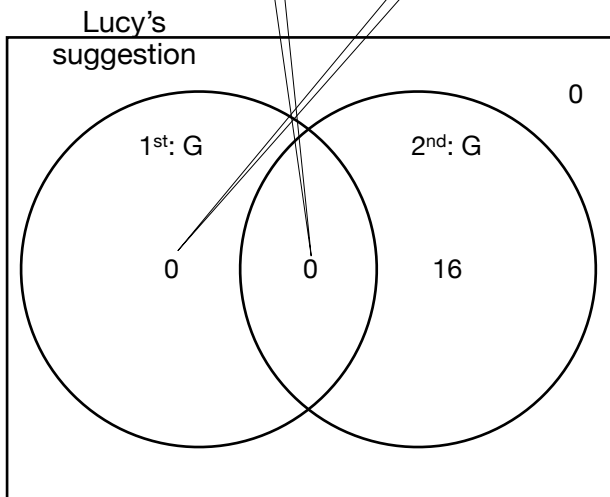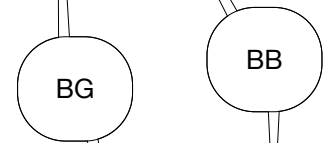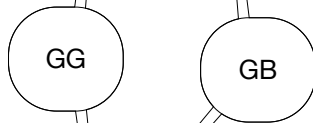| 1st choice	2nd choice	Jack?	Probability
	0 J	Yes (x2)	0
1/8 J			
	1 J´	Yes	1/8
	1/7 J	Yes	1/8
7/8 J´			
	6/7 J´	No	6/8
			1

P(Jack) = 1/8 + 1/8 = 1/4

The probabilities for Alesha getting one of the prizes are respectively 1/6, 15/64 and 1/4. Her best option is Paolo's suggestion, where she has a 1 in 4 chance of getting one of the prizes.

The probabilities for Jack getting one of the prizes are respectively 1/2, 15/64 and 1/4. His best option is Lucy's suggestion, with a 1 in 2 chance of getting one of the prizes.

2-way Tables and Venn Diagrams: expected results (where appropriate)

Lucy's suggestion		2nd choice		
		G	B	1st choice totals
1st choice	G	0	0	0
	B	16	0	16
	2nd choice totals	16	0	0

Ingrid's suggestion		2nd choice		
		G	B	1st choice totals
1st choice	G	9	3	12
	B	3	1	4
	2nd choice totals	12	4	16

GG   GB

BG   BB

Lucy's suggestion

1st: G   2nd: G

0

0   0   16

Ingrid's suggestion

1st: G   2nd: G

1

3   9   3

> It can be tricky to label the circles correctly on a Venn diagram. They correspond to the two stages of the tree diagram - so the circles in this case represent 1st choice girls and 2nd choice girls (it could equally have been boys, of course).